

Hong Kong Mathematics Olympiad (2005 – 2006)

Heat Event (Group)

香港数学竞赛 (2005 – 2006)

初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 设 a 、 b 和 c 为三个质数。若 $a < b < c$ 及 $c = a^2 + b^2$ ，求 a 的值。

Let a , b and c be three prime numbers. If $a < b < c$ and $c = a^2 + b^2$, find the value of a .

2. 若 $\log\left(\log\left(\log\left(\overbrace{100\cdots 0}^{n \text{ 个 } 0}\right)\right)\right) = 1$ ，求 n 的值。

If $\log\left(\log\left(\log\left(\overbrace{100\cdots 0}^{n \text{ zeros}}\right)\right)\right) = 1$, find the value of n .

3. 已知 $0^\circ < \theta < 90^\circ$ 及 $1 + \sin \theta + \sin^2 \theta + \cdots = \frac{3}{2}$ 。若 $y = \tan \theta$ ，求 y 的值。

Given that $0^\circ < \theta < 90^\circ$ and $1 + \sin \theta + \sin^2 \theta + \cdots = \frac{3}{2}$. If $y = \tan \theta$, find the value of y .

4. 考虑二次方程 $x^2 - (a-2)x - a - 1 = 0$ ，其中 a 为实数。设 α 和 β 为方程的根。求 a 的值使得 $\alpha^2 + \beta^2$ 的值最小。

Consider the quadratic equation $x^2 - (a-2)x - a - 1 = 0$, where a is a real number. Let α and β be the roots of the equation. Find the value of a such that the value of $\alpha^2 + \beta^2$ will be the least.

5. 已知连续 k 个正整数之和是 2006, 求 k 最大可能的值。

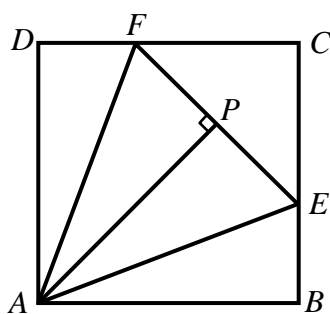
Given that the sum of k consecutive positive integers is 2006, find the maximum possible value of k .

6. 设 a 、 b 、 c 和 d 为实数且满足 $a^2 + b^2 = c^2 + d^2 = 1$ 及 $ac + bd = 0$ 。若 $R = ab + cd$, 求 R 的值。

Let a , b , c and d be real numbers such that $a^2 + b^2 = c^2 + d^2 = 1$ and $ac + bd = 0$. If $R = ab + cd$, find the value of R .

7. 如图一, 正方形 $ABCD$ 的周界是 16 cm, $\angle EAF = 45^\circ$, $AP \perp EF$ 。若 AP 的长度是 R cm, 求 R 的值。

In Figure 1, $ABCD$ is a square with perimeter equal to 16 cm, $\angle EAF = 45^\circ$ and $AP \perp EF$. If the length of AP is equal to R cm, find the value of R .



图一

Figure 1

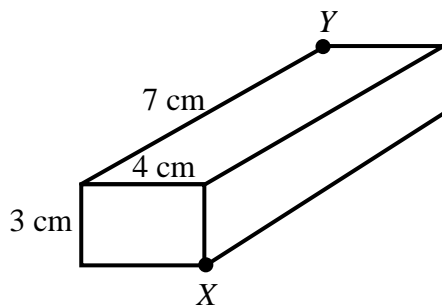
8. 已知 x 和 y 是实数且满足方程组
$$\begin{cases} \frac{100}{x+y} + \frac{64}{x-y} = 9 \\ \frac{80}{x+y} + \frac{80}{x-y} = 9 \end{cases}$$
, 若 $V = x^2 + y^2$, 求 V 的值。

Given that x and y are real numbers and satisfy the system of the equations
$$\begin{cases} \frac{100}{x+y} + \frac{64}{x-y} = 9 \\ \frac{80}{x+y} + \frac{80}{x-y} = 9 \end{cases}$$
. If

$V = x^2 + y^2$, find the value of V .

9. 如图二，一长方体盒的边长分别是 3 cm，4 cm 及 7 cm。若在盒面上从点 X 到点 Y 的最短路径的长度是 K cm，求 K 的值。

In Figure 2, given a rectangular box with dimensions 3 cm, 4 cm and 7 cm respectively. If the length of the shortest path on the surface of the box from point X to point Y is K cm, find the value of K .



图二

Figure 2

10. 已知 x 为正实数且满足不等式 $|x - 5| - |2x + 3| \leq 1$ ，求 x 的最小值。

Given that x is a positive real number which satisfy the inequality $|x - 5| - |2x + 3| \leq 1$, find the least value of x .